Stochastic processes and derivatives
Sheet 8

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Exercise 1. Let W^1, W^2 two correlated brownian motions in the risk neutral probability, with correlation coefficient ρ. We can write $W_t^1 = B_t^1$, $W_t^2 = \rho B_t^1 + \sqrt{1-\rho^2} B_t^2$, where $B := (B^1, B^2)$ is a $\mathbb{Q}$-Brownian motion. The spot rate r_t is modelized as

$$r_t \quad = \quad \alpha \; + \; Z_t^1 Z_t^2 \, ,$$

where $Z_t^1 = z_1 + W_t^1$, $Z_t^2 = z_2 + \varepsilon W_t^2$.

Part 1 – affine term structure model. We assume $\varepsilon = 0$.

1. Give the Itô's decomposition of r_t.
2. Give the shape of the forward spot rate $f(t,T)$.
3. Deduce the rate curve today according to the model parameters. What happens when $T \to \infty$?

Proof. 1. We have $r_t = \alpha + z_1 z_2 + z_2 W_t^1$ and $\mathrm{d}r_t = z_2 \, \mathrm{d}W_t^1$.

2.

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